Subert-Phong: **Example for Comparison the Accuracy the Isotopic- and the Dynamic Compactness Degrees**

(Background of BC-theory Part 7)

Place of the test section: E71 motorway in section of 188 + 660 km

Laboratory: SZTRADATESZT Ldt. - Hungary

Soil: fine silty-sand (siSa)

Statement: Measurement of water content is independent result of wet density result

A.) ISOTOPIC MEASUREMENTS

We prepare the sonde-hole for the isotopic measurement and make 21 tests by turning the instrument in the prepared sonde-hole. For each test we read and record the values of the wet density (ρ_{wet}) and the measured water content (w%).

Table 1 shows the results of the measurements with their statistical values (average, standard deviation, maximum and minimum).

N_o	w%	ρ_{wet}	N_o	w%	ρ_{wet}
1	6.60	1.883	11	7.38	1.828
2	8.06	1.843	12	7.84	1.847
3	8.10	1.851	13	7.26	1.871
4	6.09	1.843	14	7.54	1.859
5	7.70	1.854	15	6.45	1.821
6	7.14	1.870	16	7.96	1.865
7	7.38	1.852	17	9.50	1.830
8	6.99	1.829	18	8.14	1.818
9	7.24	1.898	19	8.41	1.866
10	7.01	1.853	20	7.26	1.861
			21	6.54	1.847
			average	7.46	1.852
			max	9.50	1.898
			min	6.09	1.818
			deviation	0.77	0.020

Table 1 Results of the measurement

From the 21 random results we make a three-moving-average (as if it would be a real measurement with 3 directions), attaching the end of the series onto the front of the measurement series. We also receive 21 possible results in this manner. The corrected standard deviation should be defined. With (n-1)=2 freedom degree (beside 90% probability), the student coefficient gives that, what difference is given by the result of the measurements calculated from the random data series compared to the expected value:

$$X \pm \frac{s.t_s}{\sqrt{n}}$$. where

s = standard deviation of the measurement results

n = 3, p=90% by two-sided probability (with 10% confidence interval)

t_s is the Student-coefficient, its value is 2.92 (ν=(n-1)=2 degree of freedom, and with $\alpha = 0.1$ statistical significance level)

X = average value of results

By using the above formula, we define the errors of the water content (Δw) and of the wet density ($\Delta\rho_{wet}$) which is shown in the table 2.

w%						ρ_{wet}					
single results			average	deviation	Δw	single results			average	deviation	$\Delta\rho_{wet}$
6.60	8.06	8.10	7.59	0.85	1.44	1.883	1.843	1.851	1.859	0.02	0.04
8.06	8.10	6.09	7.42	1.15	1.94	1.843	1.851	1.843	1.846	0.00	0.01
8.10	6.09	7.70	7.30	1.06	1.79	1.851	1.843	1.854	1.849	0.01	0.01
6.09	7.70	7.14	6.98	0.82	1.38	1.843	1.854	1.870	1.856	0.01	0.02
7.70	7.14	7.38	7.41	0.28	0.47	1.854	1.870	1.852	1.859	0.01	0.02
7.14	7.38	6.99	7.17	0.20	0.33	1.870	1.852	1.829	1.850	0.02	0.03
7.38	6.99	7.24	7.20	0.20	0.33	1.852	1.829	1.898	1.860	0.04	0.06
6.99	7.24	7.01	7.08	0.14	0.23	1.829	1.898	1.853	1.860	0.04	0.06
7.24	7.01	7.38	7.21	0.19	0.31	1.898	1.853	1.828	1.860	0.04	0.06
7.01	7.38	7.84	7.41	0.42	0.70	1.853	1.828	1.847	1.843	0.01	0.02
7.38	7.84	7.26	7.49	0.31	0.52	1.828	1.847	1.871	1.849	0.02	0.04
7.84	7.26	7.54	7.55	0.29	0.49	1.847	1.871	1.859	1.859	0.01	0.02
7.26	7.54	6.45	7.08	0.57	0.95	1.871	1.859	1.821	1.850	0.03	0.04
7.54	6.45	7.96	7.32	0.78	1.31	1.859	1.821	1.865	1.848	0.02	0.04
6.45	7.96	9.50	7.97	1.53	2.57	1.821	1.865	1.830	1.839	0.02	0.04
7.96	9.50	8.14	8.53	0.84	1.42	1.865	1.830	1.818	1.838	0.02	0.04
9.50	8.14	8.41	8.68	0.72	1.21	1.830	1.818	1.866	1.838	0.02	0.04
8.14	8.41	7.26	7.94	0.60	1.01	1.818	1.866	1.861	1.848	0.03	0.04
8.41	7.26	6.54	7.40	0.94	1.59	1.866	1.861	1.847	1.858	0.01	0.02
7.26	6.54	6.60	6.80	0.40	0.67	1.861	1.847	1.883	1.864	0.02	0.03
6.54	6.60	8.06	7.07	0.86	1.45	1.847	1.883	1.843	1.858	0.02	0.04
	w%	deviation	0.48				ρ_{wet}	deviation	0.01		
		average	7.46					average	1.85		
		max	8.68					max	1.86		
		min	6.80					min	1.84		

Table 2. Errors of the water content (Δw) and of the wet density ($\Delta\rho_{wet}$)

Table 3 shows the values of
$\rho_{wet-min}$, $\rho_{wet-max}$ and $w_{min}\%$, $w_{max}\%$ which are calculated by
$\rho_{wet-min} = \rho_{average} - \Delta\rho_{wet}$,
$\rho_{wet-max} = \rho_{average} + \Delta\rho_{wet}$,
$w_{min}\% = w_{average}\% - \Delta w\%$,
$w_{max}\% = w_{average}\% + \Delta w\%$

w_{max}	w_{min}	ρ_{nmax}	ρ_{nmin}
9.03	6.15	1.89	1.82
9.35	5.48	1.85	1.84
9.09	5.50	1.86	1.84
8.35	5.60	1.88	1.83
7.88	6.93	1.88	1.84
7.50	6.84	1.88	1.82
7.54	6.87	1.92	1.80
7.31	6.85	1.92	1.80
7.52	6.90	1.92	1.80
8.11	6.71	1.86	1.82
8.01	6.98	1.88	1.81
8.04	7.06	1.88	1.84
8.04	6.13	1.89	1.81
8.63	6.00	1.89	1.81
10.54	5.40	1.88	1.80
9.95	7.11	1.88	1.80
9.90	7.47	1.88	1.80
8.95	6.92	1.89	1.80
8.99	5.81	1.87	1.84
7.47	6.13	1.89	1.83
8.52	5.62	1.89	1.82

Table 3. values of $\rho_{wet\text{-}min}$, $\rho_{wet\text{-}max}$ és $w_{min}\%$, $w_{max}\%$ (the difference from average)

Four dry density values are calculated from the variation of two wet densities and two water contents (table 3.) by using the following well-known formula

$$\rho_{dry-i} = \frac{\rho_{wet-i}}{(1+w_i)},$$

the results are shown in the table 4.

	ρ_{dry}		
1.738	1.718	1.672	1.785
1.695	1.742	1.681	1.757
1.704	1.744	1.686	1.762
1.734	1.736	1.691	1.779
1.738	1.723	1.707	1.754
1.753	1.699	1.689	1.764
1.784	1.685	1.674	1.796
1.788	1.686	1.678	1.796
1.785	1.684	1.674	1.796
1.725	1.706	1.684	1.747
1.745	1.694	1.678	1.762
1.739	1.718	1.702	1.755
1.753	1.702	1.672	1.785
1.739	1.706	1.664	1.782
1.699	1.707	1.628	1.782
1.709	1.677	1.634	1.754
1.711	1.671	1.634	1.749
1.737	1.687	1.656	1.770
1.720	1.740	1.689	1.772
1.763	1.727	1.706	1.785
1.746	1.724	1.678	1.794
	deviation	0.04	
	average	1.72	
	max	1.80	
	min	1.63	

Table 4. variations of dry densities

A.1.) Accuracy of the Isotopic compactness test without consideration of the error of the Proctor density

The $T_{rg}\%$ compactness degrees are calculated by the ratio of the above calculated four values of the dry densities and the reference Proctor density ($\rho_{dry-max}$) (which is shown in the table 6.). We need to find the largest and the smallest value of the dry densities, and the half of difference between the largest and smallest value is the accuracy ($\Delta/2$), $\Delta = (T_{rgmax}\% - T_{rgmin}\%)$; which is possibly achieved with the given measurement type.

$T_{rg}\% = \rho_{dry-i} / \rho_{dry-max}$			
95%	94%	91%	98%
93%	95%	92%	96%
93%	95%	92%	96%
95%	95%	92%	97%
95%	94%	93%	96%
96%	93%	92%	96%
98%	92%	91%	98%
98%	92%	92%	98%
98%	92%	91%	98%

94%	93%	92%	95%
95%	93%	92%	96%
95%	94%	93%	96%
96%	93%	91%	98%
95%	93%	91%	97%
93%	93%	89%	97%
93%	92%	89%	96%
93%	91%	89%	96%
95%	92%	90%	97%
94%	95%	92%	97%
96%	94%	93%	98%
95%	94%	92%	98%
	average	94%	
	max	98%	
	min	89%	
	$\Delta/2$	**4.5%**	

Table 5. values of compactness degree $T_{r\rho}\%$

RESULT:
The accuracy of the STANDARD ISOTOPIC compactness test in this case is ± 4.5 $T_{rg}\%$ with probability of $p = 90\%$

A.2.) Accuracy of the isotopic compactness test in consideration of the errors of the reference density

Table 6 shows the results of the modified Proctor test: $\rho_{\text{dry-max}} = 1.83$ g/cm^3, $w_{\text{opt}} = 9.5\%$

	1	2	3	Optimum	4	5
w%	3.6	5.9	8.3	9.5	11.0	13.4
ρ_{dry}	1.71	1.78	1.83	1.83	1.82	1.77

Table 6. Results of the modified Proctor test

For calculating the error of the reference Proctor-density (largest dry density), or we prepare more Proctor (we made a large round test with more materials, more laboratories), or we accept that the accuracy of the largest density of the Proctor-curve is ca. ±0.025 g/cm^3 (or ± 3 w%). This is generally accepted as a limit without being necessary to prepare a new Proctor-test for the qualification.

The variation of the reference density in case of $\rho_{\text{drymax}} = 1.83$ g/cm^3 ±0,025 g/cm^3
$\rho_{\text{drymax-min}} = 1.805$ g/cm^3
$\rho_{\text{drymax-average}} = 1.830$ g/cm^3
$\rho_{\text{drymax-max}} = 1.855$ g/cm^3

Here the $T_{\text{rg}}\%$ compactness degrees are also calculated by the ratio of the above calculated four values of the dry densities (that values are on table 4.) and the above mentioned three reference Proctor densities. All together 252 (21x4x3) values of compactness degree will be received. They are shown in table 7.

We need to find the largest and the smallest value of the dry densities. The half of difference between the largest and smallest value is the accuracy (Δ/2),
Δ = ($T_{\text{rgmax}}\%$ − $T_{\text{rgmin}}\%$); which is possibly achieved with the given measurement type.

Trg%											
$\rho_{\text{drymax-min}}$ = 1.805				$\rho_{\text{drymax-average}}$ = 1.830				$\rho_{\text{drymax-max}}$ = 1.855			
96%	95%	93%	99%	95%	94%	91%	98%	94%	93%	90%	96%
94%	97%	93%	97%	93%	95%	92%	96%	91%	94%	91%	95%
94%	97%	93%	98%	93%	95%	92%	96%	92%	94%	91%	95%
96%	96%	94%	99%	95%	95%	92%	97%	93%	94%	91%	96%
96%	95%	95%	97%	95%	94%	93%	96%	94%	93%	92%	95%
97%	94%	94%	98%	96%	93%	92%	96%	95%	92%	91%	95%
99%	93%	93%	99%	98%	92%	91%	98%	96%	91%	90%	97%
99%	93%	93%	100%	98%	92%	92%	98%	96%	91%	90%	97%
99%	93%	93%	99%	98%	92%	91%	98%	96%	91%	90%	97%
96%	95%	93%	97%	94%	93%	92%	95%	93%	92%	91%	94%
97%	94%	93%	98%	95%	93%	92%	96%	94%	91%	90%	95%
96%	95%	94%	97%	95%	94%	93%	96%	94%	93%	92%	95%

97%	94%	93%	99%	96%	93%	91%	98%	95%	92%	90%	96%
96%	94%	92%	99%	95%	93%	91%	97%	94%	92%	90%	96%
94%	95%	90%	99%	93%	93%	89%	97%	92%	92%	88%	96%
95%	93%	91%	97%	93%	92%	89%	96%	92%	90%	88%	95%
95%	93%	91%	97%	93%	91%	89%	96%	92%	90%	88%	94%
96%	93%	92%	98%	95%	92%	90%	97%	94%	91%	89%	95%
95%	96%	94%	98%	94%	95%	92%	97%	93%	94%	91%	96%
98%	96%	94%	99%	96%	94%	93%	98%	95%	93%	92%	96%
97%	95%	93%	99%	95%	94%	92%	98%	94%	93%	90%	97%

average	94%
max	100%
min	88%
$\Delta/2$	**6.0%**

Table 7. values of compactness degrees $T_{r\rho}$%

RESULT:
The accuracy of the STANDARD ISOTOPIC compactness test in this case is ± 6.0
$T_{r\rho}$% with probability of $p = 90\%$, with the error of the reference Proctor-density

B.) DYNAMIC COMPACTNESS TEST (CWA 15846)

We measure with B&C instrument in 3 place, close to each other, in three lines, all together nine measurements (the test's area is cca.1m^2). The results of the measurements and their statistical values (average, maximum, minimum and standard deviation) are shown in the table 8.

	$T_{rE}\%$
1	97.8
2	97.6
3	98.6
4	97.1
5	97.5
6	96.3
7	98.2
8	97.8
9	97.9
average	97.6
max	98.6
min	96.3
deviation	0.653

Table 8. Result of the B&C measurement

B.1.) Accuracy of the B&C dynamic compactness test without consideration of the effect of the Proctor-curve error

With (n-1) = 1 freedom-degree (beside 90% probability), the Student-coefficient (t_s) gives that what difference is given by the result of the measurements calculated from the random data series compared to the expected value: $X \pm \dfrac{s.t_s}{\sqrt{n}}$.

Where s = standard deviation of the results
n = 2, p=90% by two-sided probability (α=10% confidence interval)
t_s is the Student-coefficient, its value is 6.314 (ν;(n-1)=2 degree of freedom with α = 0.1 significance level)
X = average value of the results

This is the difference of the expected value if we do only one measurement. You should define the minimum, maximum and average (X) of the measured on-site relative compactness rates $T_{rE}\%$.

We calculate the random results with a double-moving-average, with pre-feedback, since we average the results from two measurements according to the standard. We calculate the standard deviation (s) of the so-obtained $T_{rE}\%$ and the error of the measurement, as be shown in the table 9.

$T_{rE}\%$		
Simple results		average
97.8	97.6	97.7
97.6	98.6	98.1
98.6	97.1	97.8
97.1	97.5	97.3
97.5	96.3	96.9
96.3	98.2	97.2
98.2	97.8	98.0
97.8	97.9	97.9
97.9	97.8	97.8
	deviation	0.634
	error (±)	1.4
	average	97.6
	max	99.0
	min	96.3

Table 9. the standard deviation (s) of TrE% and the range of results

The dynamic compactness rate $T_{rd}\%$ is calculated from the formula of $T_{rd}\% = T_{rE}\% \cdot T_{rw}$, where T_{rw} is the water correction coefficient. In this case value of T_{rw} equal to 1.0 at the optimum water content, so $T_{rd}\% = T_{rE}\%$.

RESULT:
The accuracy of the realtive compactness rate with B&C test in this case is ± 1.4 $T_{rd}\%$ with probability of $p = 90\%$.
Note:in the respect of accuracy, the average of Trd% is indifferent, its interval is between 90 and 100%.

B.2.) Accuracy of the B&C dynamic compactness test in consideration of the effect of Proctor curve accuracy

The minimum, maximum and average value of coefficient of correction T_{rw} can be calculated from the Proctor-curve (±0.025 g/cm^3 or ±3 w%).
The $T_{rw\text{-}average}$ equal to 1.0 while the value of $T_{rw\text{-}min}$ and $T_{rw\text{-}max}$ is smaller than 1,0.

Values of T_{rw} are shown in table 10.

w%	ρ_{di}	T_{rw}
6.5	1.795	0.981
7.5	1.813	0.991
8.5	1.824	0.997
9.5	1.830	1.000
10.5	1.824	0.997
11.5	1.812	0.990
12.5	1.794	0.980
average		0.991
max		0.998
min		0.980
deviation		0.008
rel.deviation		1%

Table 10. values of coefficient of correction T_{rw}

After that we calculate all possible values of $T_{rd}\%$ from the formula $T_{rd}\% = T_{rE}\% \times T_{rw}$, and define the min-max-average value. Exactly values of $T_{rd}\%$ is calculated from the variations of 9 values of relative compactness-rate and 3 values of coefficient of correction that are shown in table 11. We should find the largest and the smallest value, and the half of the difference between the largest and smallest value is the accuracy ($\Delta/2$), $\Delta = (T_{rdmax}\% - T_{rdmin}\%)$, which is possibly achieved with the CWA15846 standard B&C test.

Values of $T_{rE}\%$ from table 9.	T_{rw}		
	T_{rw-max}	$T_{rw-average}$	T_{rw-min}
	1.000	0.991	0.980
97.7	97.7	96.8	95.7
98.1	98.1	97.2	96.1
97.8	97.8	96.9	95.9
97.3	97.3	96.4	95.3
96.9	96.9	96.0	94.9
97.2	97.2	96.3	95.3
98.0	98.0	97.1	96.0
97.9	97.9	97.0	95.9
97.8	97.8	97.0	95.9
		average	96.7
		max	98.1
		min	94.9
		$\Delta/2$	1.6

Table 11. values of $T_{rd}\% = T_{rE}\% * T_{rw}$

RESULT:
The accuracy of the B&C dynamic compactness test in case of consideration of the effect of the Proctor curve error is $\pm 1.6\ T_{rd}\%$.

C.) SUMMARY

The accuracy of the compactness rate measurements taken by the ISOTOPIC (nuclear) instrument and CWA15846 standard B&C dynamic one on the same experimental surface is summarized in the table 12.

Table 12. Accuracy of compactness tests at the investigated soil material

Compactness rate	ISOTOPIC measurement			Dynamic measurement (BC)		
Without consideration of the accuracy of ρ_{dmax}	$\pm$	**4.5**	$T_{r\rho}\%$	$\pm$	**1.4**	$T_{rd}\%$
In consideration of the accuracy of ρ_{dmax}	$\pm$	**6.0**	$T_{r\gamma}\%$	$\pm$	**1.5**	$T_{rd}\%$